Evaluation of the transformation models of ellipsoidal heights (h) into Orthometric height (H) for Albania

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Abstract— Several GNSS measurement campaigns have been carried out in Albania (October 1994, February 1998, September 1998) for the connection of the State Geodetic Network with the Global or European Reference System. The latest GNSS measurement campaign of Fall 2007-Spring 2008 has found the best relationship until now between the ellipsoidal heights h and so-called geoidal (natural) H, which are the basis of vertical positioning in geodesy, cartography and geoinformation. Model of transformation of h into H developed from Military Geographical Institute of Firenze, Italy used as official model in Albania, despite not being officially tested in the field. The lack of a precise gravimetric geoid for the territory of Albania has led to the impossibility of direct transformation of the ellipsoidal heights h into geoidal height H. The problem of height transformation continues to be widely addressed in the contemporary literature, where different authors propose different ways. Aim of this study is to present the different transformation models of the ellipsoidal heights h into so-called orthometric heights H of the Albanian vertical system, as well as the test results of the transformation models in different areas of Albania.

Keywords — GNSS, ellipsoidal and orthometric height, ITRF/ETRF/ALB reference, transformation.

1. Introduction

The first GPS measurement campaign in Albania carried out on period of October 6-21, 1994 by the former ITU (Military Topographic Institute, today MGIA) in collaboration with the DMAAC (United States Defense Mapping Agency Aerospace Center, now NIMA). The aim was to transform the State Geodetic Network (ALB1986) into the WGS84 (World Geodetic System 1984). 24 points of triangulation (order I/II) and 11 points of state leveling (order I) observed. Relative positions have resulted in standard deviations of up to 10 cm for (N,E,h), while standard deviations for data displacement parameters from the Krassowsky 1941 ellipsoid to WGS84 have resulted [1:D-2]:

$\sigma(N)= \pm 0.5$ m, $\sigma(E)= \pm 0.3$ m, $\sigma(h)= \pm 0.6$ m

whereas, the transformation parameters and the corresponding accuracies are [1: D-2]:

$\Delta X= 24$ m ± 4 m, $\Delta Y= -130$ m ± 4 m, $\Delta Z= -92$ m ± 4 m

The second GPS measurement campaign carried out on February 1998 by the PMU (Project Management Unit) in collaboration with the Department of Civil Engineering Geodesy of the University of Wisconsin, Florida, USA. The aim was to connect the State Geodetic Network (ALB1986) with the International Reference System (ITRF). Standard deviations for the calculated (X,Y,Z) and ellipsoidal (φ, λ, h) coordinates in WGS 84, ITRF 96, Epoch 1998 are in the range of 1-2 cm. Standard deviations for the recalculated point coordinates of the GPS campaign of October 1994, in WGS 84, ITRF 96, Epoch 1998 are in the range under of 10 cm. Standard deviations for the 2D transformation parameters (19 common points) between the coordinates in the local reference ALB1986 (Gauss-Kruger projection) and the reference WGS 84, ITRF 96, Epoch 1998 (UTM-34N projection) have resulted [2: pg. 12]:

-4-parametric linear Helmert: $\sigma_0=\pm 0.155$m, $|v|$max=0.342m.
- Polynomial transformation: $\sigma_0=\pm 0.113$m, $|v|$max=0.288 m.

For the transformation of the GPS ellipsoidal height differences (Δh) referred to WGS84 into so called orthometric height differences (ΔH), it is recommended to perform according to the equation:

$\Delta H = \Delta h - \Delta N$

where, ΔN is calculated referring to the geoidal model EGM96, but not knowing the difference between the vertical reference data of Albania and the geoid EGM96 the transformation model is always recommended to be used in relative sense in plain and hilly areas [2: pg. 15, Volume 1]. No information given regarding the accuracy of the transformation of the ellipsoidal heights h referring to the WGS 84 ellipsoid to the so-called orthometric height H.

In the framework of the CRODYN project, on September 1998, the third GPS measurement campaign carried out by the former ITU (Military Topographic Institute, today MGIA) in cooperation with the Federal Agency for Cartography and Geodesy of Germany. The purpose of this GPS measurement campaign was to connect the State Geodetic Network (ALB1986) with the European Reference System ETRS89 (ETRF1989). After processing, the standard deviations for the point coordinates in ITRF96, Epoka1998.7 resulted in ± 2 mm in plan and ± 6.5 mm in height. For the transformation of coordinates from the global reference ITRF96 to the European reference ETRS89, the 7-parameters are [3: pg. 112]:

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T_X = 4.1 cm, T_Y = 4.1 cm, T_Z = -4.9 cm, D=10-8, R_X = 0.20 (0.0012/year), R_Y = 0.50 (0.0012/year), R_Z =-0.65 (0.0012/year)

where, (T_X,T_Y,T_Z) are the transformation parameters from ITRF96 to ETRS89, (R_X,R_Y,R_Z) are the rotation parameters from ITRF96 to ETRS89.

In the period autumn of 2007 - spring 2008, the fourth GNSS measurement campaign is carried out by IGUS (Military Geographical Institute of Albania) in cooperation with the Military Geographical Institute of Florence (IGUF), Italy. The aim was to find the transformation parameters from the European reference system (ETRF2000, Epoch 2008) to local (ALB1986). GNSS surveys were carried out at approximately 150 points in the state triangulation and leveling networks (Fig. 1). The coordinates of the points are calculated in ITRF2005 and transformed into ETRF2000, Epoch2008.0 [4 : pg. 18].

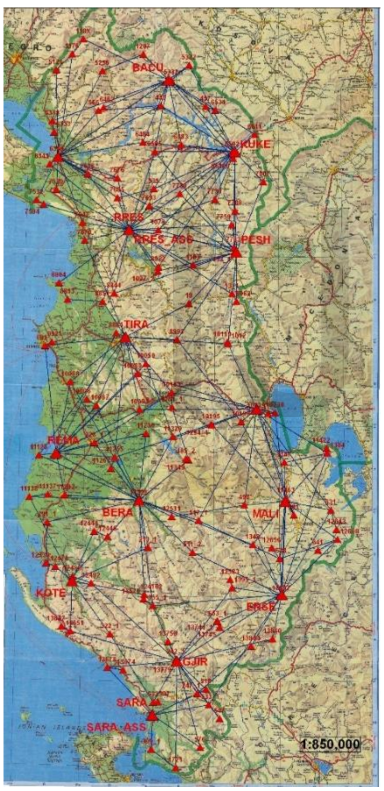


Figure 1: Scheme of the points measured autumn 2007 - spring 2008 [4]

TABLE 1: 7P HELMERT FOR HEIGHT TRANSFORMATION OF $H_{GRS80, EGM2008}$ INTO $H_{ALB1987}$

T_X(m)	T_Y(m)	T_Z(m)	S(ppm)	R_X (")	R_Y (")	R_Z (")
5.792	-13.506	-0.902	0.0611	0.2800	0.1440	-0.3581

For the transformation of heights hGRS80, EGM2008 into HALB87, based on 125 common points, 7P Helmert are calculated in advance (Tab. 1) with "start" system ETRF2000, Epoka2008 (with height hGRS80 and H EGM2008) and "arrival" system again ETRF8000, Epoch 2008 with true geoid heights (ALB86 leveling) [4 : pg. 18].

For grid vertices (Fig. 2), with cell dimensions 2'.5x2'.5 (Fig. 3) of the ellipsoidal zone between the parallels 39°÷43°.5 and the meridians 18°÷22°, locally adapted to the above 7-parameters of the Helmert transformation with the model of the global geoid EGM2008, the geoidal waves N for the Albanian territory have been calculated.

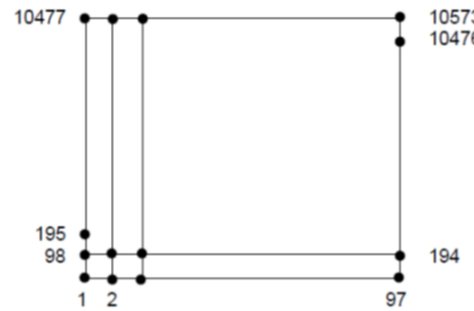


Figure. 2: Matrix-shaped grids (103x97=10753)

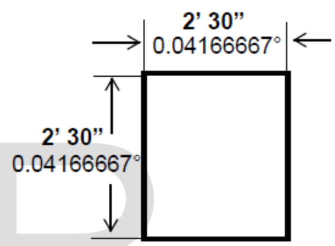


Figure 3: Grid dimensions

The calculation of the geoidal heights of any point (not grid vertices) is done through a bilinear interpolation of the waves (z) within the 4 vertices of the grid cell (Fig. 4), with a polynomial of the form:

$$z = (1-s)\,(1-t)\,z_{00} + t\,(1-s)\,z_{01} + s\,(1-t)\,z_{10} + s\,t\,z_{00}$$

where:

$$x - x_0/x_1 - x_0 = s,\quad y - y_0/y_1 - y_0 = t$$

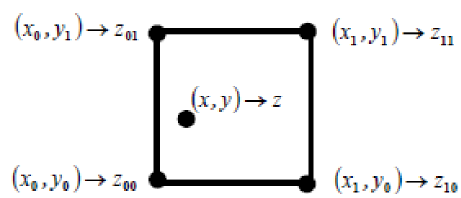


Figure 4: Grid cell

For each point of the territory of Albania, the geoid height (HALB86) is calculated from the respective ellipsoidal height (hGRS80) according to the relation:

$$H_{ALB1987} = h_{GRS80} + N_{(EGM2008-GRS80)}$$

The transformation of ellipsoidal to geoidal heights is realized through the ALBGEO3 software. According to [4], the average quadratic deviations of the calculation of heights H referred to ALB1987 from the ellipsoidal heights h referred to GRS80 have resulted: σh→H=±20cm (confidence level 68%), σh→H=±40cm (confidence level 85%).

2. Materials and Methods

2.1 The linear interpolation polynomial proposed

Based on the 3D coordinates (φ,λ,h) [4] of n common points and the so-called orthometric heights H, the transformation of the ellipsoidal heights h referring GRS80 into the orthometric height H referred to ALB1987 is realized through a linear interpolation polynomial of degree 3 [5: pg. 86], with three variables (x,y,z), proposed by the Department of Geodesy, Faculty of Civil Engineering, of the form:

$$P_3^3(x,y,z)=a_1+a_2y+a_3x+a_4z+a_5yx+a_6yz+a_7xz$$
$$+a_8y^2+a_9x^2+a_{10}z^2+a_{11}xyz+a_{12}y^3++a_{13}yx^2+a_{14}yz^2$$
$$+a_{15}y^2x+a_{16}x^3+a_{17}xz^2+a_{18}y^2z+a_{19}x^2z+a_{20}z^3$$

where, x=dφi =φi – φ_0, y=dλi =λi – λ_0, z=dhi =hi – h_0 are the corresponding differences between the latitudes, longitudes, and ellipsoidal heights of the points in common in both systems. The transformed height in the ALB1987 given as:

$$H_{ALB1987}= h_{GRS80}+P^3_m\ (x,y,z)$$

where, m is the degree of the linear interpolation polynomial.

Once the barycenters have been calculated (φ_0, λ_0, h0) as φ_0=Σφi/n = 0.717224124 rad, λ_0=Σλi/n= 0.350605742 rad, h_0=Σhi/n= 637.7247m, the system of vector equations with reduced coordinates for n common points has the form:

$$\begin{bmatrix} 1 & d\varphi_1 & d\lambda_1 & dh_1 & d\varphi_1 d\lambda_1 & \cdots & dh_1^3 \\ 1 & d\varphi_2 & d\lambda_2 & dh_2 & d\varphi_2 d\lambda_2 & \cdots & dh_2^3 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 1 & d\varphi_{103} & d\lambda_{103} & dh_n & d\varphi_n d\lambda_n & \cdots & dh_n^3 \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_{20} \end{bmatrix} = \begin{bmatrix} H^1_{ALB86}-h^1_{GRS80} \\ H^2_{ALB86}-h^2_{GRS80} \\ \vdots \\ H^{103}_{ALB86}-h^{103}_{GRS80} \end{bmatrix}$$

or in matrix form: $A_{n,20}\cdot X_{20,1} = L_{n,1}$, and from the solution of vector equations with reduced coordinates according to the simplified method of the minimum squares ([vv]→min), the polynomial coefficients are found: a1, a2, a3, ..., a19, a20:

$$X = (AT \cdot A)-1\ \cdot (AT \cdot L), \qquad vi = (A\ \cdot X) - L$$

The average quadratic deviation σ_0 calculated as σ_0=±SQRT([vv]/n) in the transformation of ellipsoidal heights h referred to GRS80 to orthometric heights H referred to ALB1987 has resulted: σ_0=±0.307 m.

In the case of the transformation polynomial of degree 4 with three variables [5: pg. 87]:

$$P_4^3(x,y,z)=P_3^3(x,y,z)+a_{21}x^4+a_{22}x^3y+a_{23}x^3z+a_{24}x^2y^2$$
$$+a_{25}x^2z^2+a_{26}y^4+a_{27}y^3x+a_{28}y^3z+a_{29}z^4+a_{30}z^3x+a_{31}z^3y$$
$$+a_{32}z^2xy+a_{33}zx^2y+a_{34}zxy^2$$

or in matrix form: $A_{n,\ 34}\cdot X_{34,\ 1}$=Ln, 1, and from the solution of vector equations with reduced coordinates according to the simplified method of the minimum squares ([vv]→min), the polynomial coefficients are found: a1, a2, a3, ..., a33, a34. The average quadratic deviation in the transformation of ellipsoidal heights h referred to GRS80 into orthometric heights H referred to ALB1987 has resulted: σ_0=±0.226 m.

In the case of the transformation polynomial of degree 5 with three variables [5: pg. 87]:

$$P_5^3(x,y,z)=P_4^3(x,y,z)+a_{35}x^4y+a_{36}x^4z+a_{37}x^3yz+a_{38}x^3z^2$$
$$+a_{39}x^2y^2z+a_{40}x^2yz^2+a_{41}x^5+a_{42}y^4x+a_{43}y^4z+a_{44}y^3xz$$
$$+a_{45}y^2xz^2+a_{46}y^5+a_{47}z^4x+a_{48}z^4y+a_{49}z^3xy+a_{50}z^5$$
$$+a_{51}x^3y^2+a_{52}y^3x^2+a_{53}y^3z^2++a_{54}z^3x^2+a_{55}z^3y^2$$

or in matrix form: $A_{n,55}\ \cdot X_{55,1}=L_{n,1}$, and from the solution of vector equations with reduced coordinates according to the simplified method of the minimum squares ([vv]→min), the polynomial coefficients are found: a1, a2, a3, ..., a54, a55. The average quadratic deviation in the transformation of ellipsoidal heights h referred to GRS80 into orthometric heights H referred to ALB1987 has resulted: σ_0=±0.1567m.

The errors of the transformation of ellipsoidal heights hGRS80 into orthometric HALB1987 through ALBGEO3 model estimated: ±20 cm (at confidence level of 68%) and ±40 cm at the confidence level of 95% [4: pg. 39, 3rd part].

The errors of the transformation of ellipsoidal heights h (GRS80) into orthometric H (ALB1987) through the polynomial model found by the Department of Geodesy, Faculty of Civil Engineering for the transformation of ellipsoidal heights h (GRS80) into orthometric H (ALB1987) estimated [5: pg. 89]: 0÷10 cm 55% of cases, ÷20 cm 80%, ÷30 cm 92%, ÷40 cm 98%, >40 cm 2%.

3. RESULTS

3.1 GNSS Field Survey

For the real control of the transformation model treated above, static GNSS satellite measurements performed, in four different areas (Fig. 5), at the points referred ALB1987, which not used to derive the transformation models in plan and height during the autumn 2007-spring 2008 GNSS campaign.

Figure 5: Test areas of transformation models of h to H (illustrated by Google.com)

The Tables 2, 3, 4, 5, 6 give the 3D coordinates (φ,λ,h) and (N,E,h) in the reference ETRF2000, Epoch2021.5, respectively for the areas: (a) Berat, (b) Tepelena, (c) Shkodra, (d) Tirana and (e) Kukes-Peshkopi.

TABLE.2: COORDINATES OF POINTS (Φ, Λ, H) MEASURED IN THE BERAT AREA (ETRF2000, EPOCH2021.5)

No.	Catalogue No.	ϕ_{GRS80} *(dd.mmssssss)*	Λ_{GRS80} *(dd.mmssssss)*	N(UTM) (m)	E(UTM) (m)	h_{GRS80} *(m)*
1	124102	40.20143351	19.58410227	4465698.118	413197.659	396.109
2	12495	40.2219519	19.59384661	4469542.423	414596.896	351.93
3	12494	40.22220782	19.56300923	4469673.174	410155.525	498.424
4	12488	40.2402535	19.57356906	4472752.258	411738.915	709.341
5	12574	40.25122421	19.59560811	4474863.366	415072.611	632.069
6	12477	40.26345737	19.55393890	4477472.939	409054.295	842.152
7	12471	40.28008705	19.57466647	4480097.995	412083.878	736.033
8	12470	40.28016748	19.55003796	4480169.824	408168.270	896.533
9	12563	40.29100403	20.01017197	4482178.214	416701.043	884.696
10	12461	40.30118997	19.55355413	4484175.072	409045.153	950.894
11	12453	40.31524148	19.5522422	4487278.143	408774.225	1029.756
12	12450	40.32371241	19.5942460	4488584.478	414907.823	910.296
13	12448	40.34081603	19.57019816	4491435.518	411166.401	750.701
14	12522	40.34333564	20.01065042	4492146.143	416924.677	935.317
15	12431	40.36394722	19.54435582	4496140.653	407969.200	988.542
16	12423	40.38469466	19.58102194	4500012.850	412871.565	656.799
17	11298	40.4227665	19.56427155	4506843.073	410898.076	276.159

TABLE 3: COORDINATES OF POINTS (Φ, Λ, H) MEASURED IN THE TEPELENA AREA (ETRF2000, EPOCH2021.5)

No.	Catalogue No.	ϕ_{GRS80} *(dd.mmssssss)*	Λ_{GRS80} *(dd.mmssssss)*	N(UTM) (m)	E(UTM) (m)	h_{GRS80} *(m)*
1	12444	40.34597428	19.42581380	4493288.856	391347.586	638.864
2	13745	40.12439585	20.21571331	4451503.764	446038.629	613.000
3	518	39.58092635	20.17556353	4424578.555	440117.606	644.893
4	576	39.44244274	20.16120659	4399168.328	437453.094	492.205

5	535	41.55478862	20.00103364	4642481.469	417329.229	318.906
6	566	42.14025747	19.42169868	4676575.013	393120.968	393.313
7	631	40.40259895	20.56068384	4502558.360	494526.228	887.526
8	1103	41.37583967	20.12510392	4609315.714	434547.091	851.367

TABLE 4: COORDINATES OF POINTS (Φ, Λ, H) MEASURED IN THE SHKODRA AREA (ETRF2000, EPOCH2021.5)

No.	Catalogue No.	ϕ_{GRS80} *(dd.mmssssss)*	Λ_{GRS80} *(dd.mmssssss)*	N(UTM) (m)	E(UTM) (m)	h_{GRS80} *(m)*
1	6343	42.00130142	19.22569567	4651443.067	366047.193	72.0951
2	6344	42.00400046	19.25089825	4652218.780	369099.761	75.2734
3	6407	42.04096554	19.36315503	4658412.345	384905.152	254.583
4	6440	42.03019511	19.33504968	4656385.359	381168.975	180.236
5	6447	42.00391688	19.32222729	4652015.927	379065.759	90.7896
6	6592	42.0437555	20.24554509	4658501.777	451643.469	413.698

TABLE 5: COORDINATES OF POINTS (Φ, Λ, H) MEASURED IN THE TIRANA AREA (ETRF2000, EPOCH2021.5)

No.	Catalogue No.	ϕ_{GRS80} *(dd.mmssssss)*	Λ_{GRS80} *(dd.mmssssss)*	N(UTM) (m)	E(UTM) (m)	h_{GRS80} *(m)*
1	1000	41.20042123	19.49366521	4576554.990	401835.713	177.1826
2	8884	41.21476314	19.48571265	4579756.784	400960.544	215.243
3	8879	41.22161222	19.50509873	4580599.753	403617.604	342.7391
4	10027	41.19120288	19.51258525	4574911.843	404352.729	268.6779
5	10037	41.17120855	19.5350246	4571169.550	407662.706	351.9794
6	8886	41.2039047	19.42346694	4577768.539	392043.246	267.9903
7	8814	41.21385236	19.46261848	4579524.660	397449.672	136.1983
8	8884	41.21476249	19.48571236	4579756.584	400960.474	215.546
9	8922	41.37274472	20.01555257	4608515.453	419369.782	324.88
10	8962	41.29270702	20.26248679	4593399.933	453274.305	871.428

TABLE 6: COORDINATES OF POINTS (Φ, Λ, H) MEASURED IN THE KUKËS-PESHKOPI AREA (ETRF2000, EPOCH2021.777)

No.	Catalogue No.	ϕ_{GRS80} *(dd.mmssssss)*	Λ_{GRS80} *(dd.mmssssss)*	N(UTM) (m)	E(UTM) (m)	h_{GRS80} *(m)*
1	65106	42.02292099	20.22005952	4654571.978	447596.463	869.624
2	7719	41.58167456	20.23482109	4646767.706	450015.691	819.813
3	7733	41.53288717	20.1926822	4637934.238	443929.738	669.522
4	7734	41.55294012	20.21382178	4641628.290	446985.351	583.533
5	7737	41.53384292	20.21579048	4638202.429	447413.491	1144.176
6	7756	41.49196705	20.20044801	4630241.891	444737.974	912.250
7	7760	41.47490996	20.22137399	4627426.163	447699.410	887.960
8	7770	41.44374641	20.23238213	4621504.406	449274.831	817.818
9	7771	41.43580008	20.20032361	4620321.695	444632.474	556.490
10	7781	41.42500931	20.22502804	4618198.606	448476.306	592.972
11	7782	41.42217782	20.20363187	4617348.310	445374.084	502.141
12	7784	41.40509803	20.20573127	4614544.426	445838.114	557.267
13	7785	41.412449	20.22236531	4615563.063	447841.794	544.573
14	8835	41.34094302	19.4612022	4602686.786	397449.838	799.953

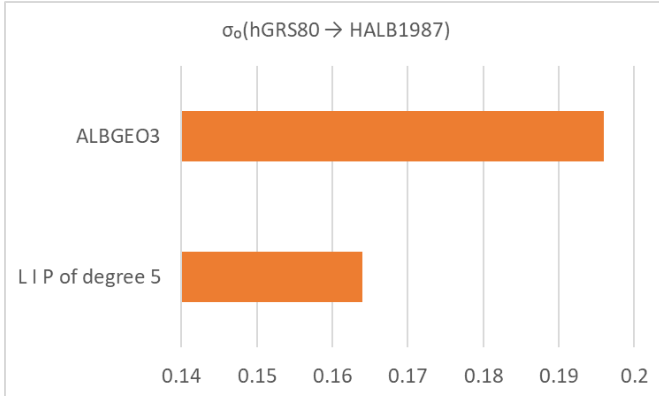
15	8836	41.33492833	19.43366496	4602117.622	393842.246	308.562
16	8899	41.34045815	19.59156193	4602301.592	415596.132	477.140
17	88105	41.37320238	19.59151338	4608699.195	415659.948	340.523
18	8903	41.35133251	20.28153169	4604061.964	455900.730	614.967
19	8921	41.37288672	20.10152014	4608438.782	430932.727	1068.836
20	8922	41.37274471	20.0155525	4608515.439	419369.764	325.000
21	8927	41.37124013	20.26198684	4607751.081	453251.612	752.754
22	8929	41.35421878	20.02560716	4605253.683	420735.033	378.156
23	8937	41.35093003	20.26426158	4603951.317	453753.608	608.840

3.2 Comparison between coordinates of different kinds

In the Tables 7, 8, 9, 10, 11 given the transformed heights from reference ETRF2000, Epoch2021.5 into reference ALB1987 through two transformation models: (1) ALBGEO3 software and (2) linear interpolation polynomial (LIP) of degree 5, as well as the corresponding quadratic mean deviations σ_{01} and σ_{02} for the areas: (a) Berat, (b) Tepelena, (c) Shkodra, (d) Tirana and (e) Kukes-Peshkopi.

TABLE 7: TRANSFORMED HEIGHTS hGRS80 → HALB1987 FOR THE MEASURED POINTS IN THE AREA OF BERAT

No.	HALB1987 (m)	hGRS80 → HALB1987 (LIP of degree 5)	V1	hGRS80 → HALB1987 (ALBGEO3)	V2
1	361.036	360.946	0.090	361.189	-0.153
2	316.76	316.666	0.094	316.931	-0.171
3	463.543	463.282	0.261	463.348	0.196
4	673.94	674.091	-0.151	673.861	0.079
5	596.512	596.61	-0.098	596.562	-0.050
6	806.62	806.846	-0.226	806.482	0.138
7	700.55	700.51	0.040	700.381	0.169
8	861.1	861.171	-0.071	860.771	0.329
9	848.97	848.955	0.015	848.667	0.303
10	915.2	915.365	-0.165	914.968	0.232
11	994.05	994.186	-0.136	993.646	0.404
12	874.436	874.535	-0.099	874.165	0.271
13	714.74	715.039	-0.299	714.893	-0.153
14	899.326	899.491	-0.165	899.001	0.325
15	952.7	952.986	-0.286	952.444	0.256
16	620.997	621.121	-0.124	620.913	0.084
17	240.476	240.59	-0.114	240.770	-0.294
			σ_{01} = 0.164 m		σ_{02} = 0.196 m

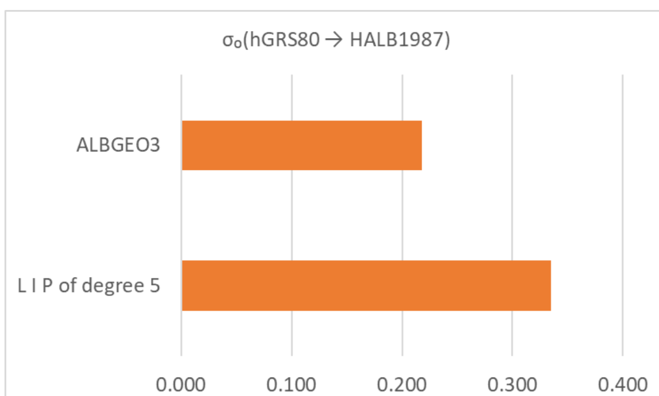


The errors of the transformation of the ellipsoidal heights h (GRS80) into orthometric H (ALB1987) at the measured points of Berat area for the transformation model of the heights (1) and (2), are respectively:

(1) 0÷10.0cm 41% of cases, ÷20.0cm 76.5%, ÷30.0cm 100%,
(2) 0÷10.0cm 18% of cases, ÷20.0cm 53%, ÷30.0cm 82.5%, ÷40.0cm 100%.

TABLE 8: TRANSFORMED HEIGHTS FOR THE MEASURED POINTS IN THE AREA OF TEPELENA

No.	HALB1987 (m)	hGRS80 → HALB1987 (LIP of degree 5)	V1 (m)	hGRS80 → HALB1987 (ALBGEO3)	V2 (m)
1	603.997	604.026	-0.029	604.064	-0.067
2	577.522	577.33	0.192	577.332	0.190
3	610.94	611.076	-0.136	611.036	-0.096
4	459.91	460.015	-0.105	460.441	-0.531
5	278.071	278.222	-0.151	278.185	-0.114
6	353.04	353.153	-0.113	352.910	0.130
7	847.179	847.009	0.170	847.300	-0.121
8	810.035	810.123	-0.088	810.094	-0.059
			σ_{01}= 0.335 m		σ_{02} = 0.218 m

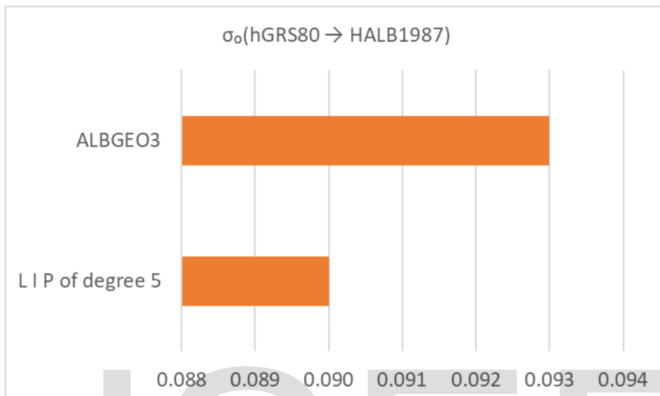


The errors of the transformation of the ellipsoidal heights h (GRS80) into orthometric H (ALB1987) at the measured points of Tepelena area for the transformation model of the heights (1) and (2), are respectively:

(1) 0÷10.0cm 37.5% of cases, ÷20.0cm 100%,
(2) 0÷10.0cm 50% of cases, ÷20.0cm 87.5%, >40.0cm 100%.

TABLE 9: TRANSFORMED HEIGHTS HGRS80 → HALB1987 FOR THE MEASURED POINTS IN THE AREA OF SHKODRA

No.	HALB1987 (m)	hGRS80 → HALB1987 (LIP of degree 5)	V1 (m)	hGRS80 → HALB1987 (ALBGEO3)	V2 (m)
1	34.95	34.81	0.140	34.877	0.073
2	37.93	37.89	0.040	37.964	-0.034
3	216.101	216.198	-0.097	216.046	0.055
4	140.399	142.197	-1.798	142.081	-1.682
5	55.62	53.073	2.547	53.051	2.569
6	370.474	370.434	0.040	370.317	0.157
			σ_{01}= 0.090 m		σ_{02} = 0.093 m

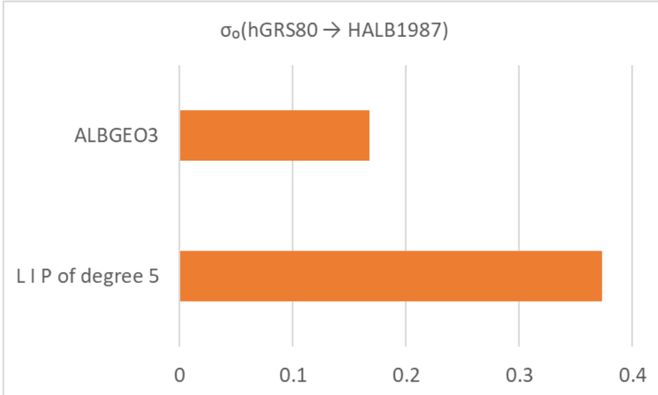


The errors of the transformation of the ellipsoidal heights h (GRS80) into orthometric H (ALB1987) at the measured points of Shkodra area for the transformation model of the heights (1) and (2), are respectively:

(1) 0÷10.0cm 50% of cases, ÷20.0cm 66.7%, >40.0cm 100%,
(2) 0÷10.0cm 50% of cases, ÷20.0cm 66.7%, >40.0cm 100%.

TABLE 10: TRANSFORMED HEIGHTS HGRS80 → HALB1987 FOR THE MEASURED POINTS IN THE AREA OF TIRANA

No.	HALB1987 (m)	hGRS80 → HALB1987 (LIP of degree 5)	V1 (m)	hGRS80 → HALB1987 (ALBGEO3)	V2 (m)
1	141.201	140.698	0.503	140.921	0.280
2	179.152	178.693	0.459	179.059	0.093
3	306.532	305.881	0.651	306.430	0.102
4	233.29	232.022	1.268	232.408	0.882
5	315.487	315.162	0.325	315.538	-0.051
6	232.81	232.168	0.642	232.853	-0.043
7	100.283	99.957	0.326	100.124	0.159
8	179.152	178.996	0.156	179.363	-0.211
9	285.45	285.464	-0.014	285.577	-0.127
10	829.805	829.856	-0.051	829.610	0.195
			σ_{01}= 0.373 m		σ_{02} =0.168 m

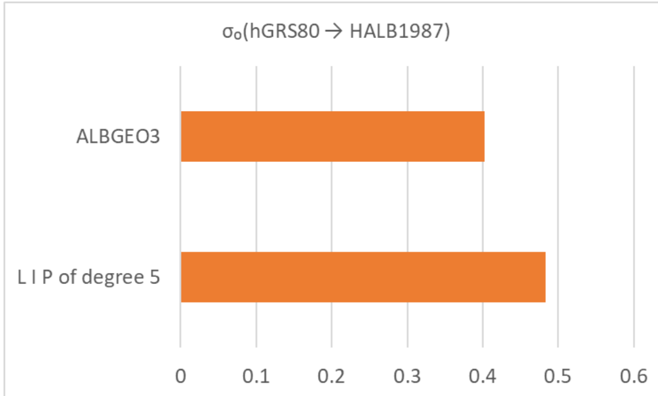


The errors of the transformation of the ellipsoidal heights h (GRS80) into orthometric H (ALB1987) at the measured points of Tirana area for the transformation model of the heights (1) and (2), are respectively:

(1) 0÷10.0cm 20% of cases, ÷20.0cm 30%, ÷30.0cm 40%, ÷40.0cm 50%, >40.0cm 100%,
(2) 0÷10.0cm 50% of cases, ÷20.0cm 80%, ÷30.0cm 90%, >40.0cm 100%.

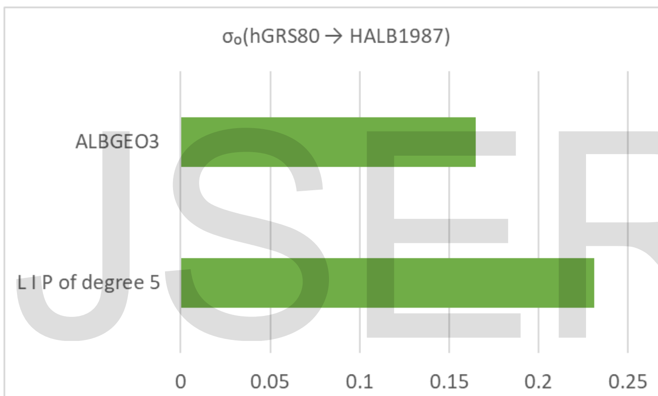
TABLE 11: TRANSFORMED HEIGHTS HGRS80 → HALB1987 FOR THE MEASURED POINTS IN THE AREA OF KUKËS-PESHKOPI

No.	HALB1987 (m)	hGRS80 → HALB1987 (polinomi i shkallës 5)	V1	hGRS80 → HALB1987 (ALBGEO3)	V2
1	826.900	826.5796	0.320	826.632	0.268
2	777.341	776.8248	0.516	776.832	0.509
3	627.045	626.8323	0.213	627.17	-0.125
4	539.220	540.6605	-1.440	540.871	-1.651
5	1101.565	1101.5056	0.059	1101.618	-0.053
6	869.830	869.7589	0.071	870.158	-0.328
7	845.500	845.4375	0.063	845.747	-0.247
8	775.774	775.3787	0.395	775.684	0.090
9	515.034	514.3891	0.645	514.763	0.271
10	550.903	550.7216	0.181	551.063	-0.160
11	460.237	460.1679	0.069	460.478	-0.241
12	515.502	515.299	0.203	515.653	-0.151
13	502.655	502.4835	0.171	502.813	-0.158
14	762.660	763.7265	-1.067	762.648	0.012
15	271.907	272.4623	-0.555	271.707	0.200
16	437.934	438.5082	-0.574	438.137	-0.203
17	301.262	301.6189	-0.357	301.492	-0.230
18	572.840	572.8182	0.022	572.879	-0.039
19	1027.745	1027.8241	-0.079	1027.919	-0.174
20	285.449	285.6973	-0.248	285.584	-0.135
21	710.653	710.5546	0.098	710.737	-0.084
22	338.404	338.846	-0.442	338.641	-0.237
23	566.881	566.7907	0.090	566.945	-0.064
			σ_{01}= 0.483m		σ_{02}=0.402m



The errors of the transformation of the ellipsoidal heights h (GRS80) into orthometric H (ALB1987) at the measured points of Kukes-Peshkopi area for the transformation model of the heights (1) and (2), are respectively:
(1) 0÷10 cm 26% of cases, ÷ 20 cm 61%, ÷ 30 cm 92%, ÷ 40 cm 92%, > 40 cm 8%,
(2) 0÷ 10.0 cm 35% of cases, ÷ 20 cm 52%, ÷ 30 cm 57%, ÷ 40 cm 74%, > 40 cm 26%.

For all points (64 points): σ01= 0.231 m, σ02=0.165 m.



4. Conclusions

1. The transformation model proposed by the Department of Geodesy compared to the ALBGEO3 model developed by MGIF has the following differences and advantages:
• no need to use of the global geoid EGM08 and its adaptation to the territory of Albania.
• no need to create the grids for calculating of transformed heights at points, which are not its vertices.
• The model is easily conceivable both from a theoretical and practical point of view.
• achieves higher accuracy of altitude transformation compared to the ALBGEO3 model.
• the possibility of further improving it in terms of accuracy, with the future addition of satellite observations to existing vertical reference points, which the ALBGEO3 model does not allow because their computational program is not source, but EXE.

2. Both models of transformation of h to H, which not officially approved recommended to be used as a vertical control reference for the topographic mapping of the territory of Albania up to a scale of 1: 5000.

3. Both models of transformation of h to H can be improved if we will perform additional measurements in existing points of the horizontal control network (the height of which is determined by geometric levelling) referred ALB1986 and vertical network referred ALB1987.

5. References

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